Extraction of gassy sediments: the bubble pressure Correction

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1 - We are considering a sample with hard grains and low cohesion: we are in the Terzaghi approximation and then $b = 1$ and $N = \infty$.

2 - Using the poromechanics relation:

$$
\begin{aligned}
d(\sigma + p) &= d(K\epsilon - bp + p) \approx d(K\epsilon) \\
d\varphi &= bd\epsilon + \frac{dp}{N} \approx d\epsilon \\
\Rightarrow \quad d(\sigma + p) &= Kd\varphi
\end{aligned}
$$

The relation for the fluid gives us:

$$
\begin{aligned}
\frac{dp_F}{K_F} &= \frac{d\rho_F}{\rho_F} \\
dp_F &= K_F \frac{d\rho_F}{\rho_F}
\end{aligned}
$$

Using now the undrained condition,(the quantity of fluid is constant), $d(\rho_F \phi) = 0$:

$$
\begin{aligned}
\phi d\rho_F + \rho_F d\phi &= 0 \\
d\varphi &\approx -\phi_0 \frac{dp_F}{K_F}
\end{aligned}
$$

Combining the previous equations we finally obtain:

$$
\begin{aligned}
dp &= Kd\varphi - d\sigma \\
&= -\phi_0 K \frac{dp}{K_F} - d\sigma \\
&= -\frac{d\sigma}{1 + \phi_0 \frac{K}{K_F}} \\
d(\sigma + p) &= \frac{\phi_0 \frac{K}{K_F}}{1 + \phi_0 \frac{K}{K_F}} d\sigma
\end{aligned}
$$

The sample fails when $p > -\sigma$. The final equations shows that in the current conditions, this is never the case. Classical undrained poroelasticity cannot explain the failure of the sample.

3 - The liquid in the sample is containing dissolved gas. When the confining pressure is decreased, the liquid pressure is also decreased, and the solution is out of equilibrium and starts degasing. This is controlled by Henry's law: $p_J = x_J K_H$ with J the dissolved gas.

4 - Be careful, the assumption of the equality of pressure for gas and liquid is usually wrong but it is made here for the sake of simplicity. However, we will see that in the case of bubbles larger than 1μm, this assumption is valid.

$$
\begin{aligned}
\frac{dp}{K_{mix}} = \frac{d\rho_{mix}}{\rho_{mix}} &= -\frac{dV_{mix}}{V_{mix}} = -\frac{dV_L}{V_{mix}} - \frac{dV_G}{V_{mix}} \\
&= -\frac{V_L}{V_{mix}}\frac{dV_L}{V_L} - \frac{V_G}{V_{mix}}\frac{dV_G}{V_G} \\
&= -\left(1-\nu\right)\frac{dV_L}{V_L} - \nu\frac{dV_G}{V_G} \\
&= \left(1-\nu\right)\frac{dp}{K_L} + \nu\frac{dp}{p} \\
\Rightarrow \frac{d\rho_{mix}}{\rho_{mix}} = -\frac{dV_{mix}}{V_{mix}} &= \left(\frac{1-\nu}{K_L} + \frac{\nu}{p}\right) dp
\end{aligned}
$$

(considering that the gas is perfect)

Now, if we consider again the undrained condition, the conservation equations give:

$$
\begin{aligned}
\rho_{mix} V &= \rho_{mix0} V_0 \\
\rho_G \nu V &= \rho_{G0} \nu_0 V_0 \\
\rho_L \left(1-\nu\right) V &= \rho_{L0}\left(1-\nu_0\right) V_0
\end{aligned}
$$

Replacing then $\frac{V}{V_0}$ by $\frac{\rho_{mix0}}{\rho_{mix}}$ eliminating ν and considering that $\rho_{L0} \approx \rho_L$, we obtain the desired equation:

$$
\frac{d\rho_{mix}}{\rho_{mix}} \approx \frac{\rho_{mix}}{\rho_{mix0}}\left(\frac{1-\nu_0}{K_L} + \frac{\nu_0 p_0}{p^2}\right) dp
$$

Actually, the conservation of matter has to take into account that when the pressure varies the quantity of gas differs because of Henry's law. The new term refers to the change of gas quantity when the pressure changes. By definition, the fluid bulk modulus is: $\frac{dp}{K_F} = \frac{d\rho_{mix}}{\rho_{mix}}$

5 - We use again the equation for poroelasticity:

$$
\begin{aligned}
d\varphi &= d\epsilon \qquad d\sigma = d\left(K\epsilon - p\right) \\
d\varphi &= \frac{1}{K}d\sigma + \frac{1}{K}dp
\end{aligned}
$$

6 - We have still $d\varphi = -\phi_0 \frac{dp}{K_F}$,

$$
\begin{aligned}
-\frac{d\sigma}{K\phi_0} &= \left(\frac{1}{\phi_0 K} + \frac{1}{K_F}\right) dp \\
&= dp\left(\frac{1}{\phi_0 K} + \frac{\nu_0 p_0}{p^2} + \frac{1-\nu_0}{K_L}\right) + \alpha d\left(\frac{x-x_0}{p}\right) \\
&= dp\left(\beta + \frac{\nu_0 p_0}{p^2}\right) + \alpha d\left(\frac{x-x_0}{p}\right) \\
-\frac{1}{K\phi_0}\left(\sigma - \sigma_0\right) &= \left(p - p_0\right)\left(\beta + \frac{\nu_0}{p}\right) + \alpha\frac{x-x_0}{p}
\end{aligned}
$$

7 - Directly after unloading, the pressure follows the decrease of the confining pressure ($x_+ = x_0$):

$$
-\frac{1}{K\phi_0}\left(\sigma_+ - \sigma_0\right) = \left(p_+ - p_0\right)\left(\beta + \frac{\nu_0}{p_+}\right)
$$

The instantaneous responds does not then depend if the pore pressure is below or above the bubble pressure.

8 - After a long time, the liquid starts to desaturate and the quantity of gas increases. We have because of Henry's law, $x_\infty = p_\infty / K_H$

$$-\frac{1}{K\phi_0}\left(\sigma_+ - \sigma_0\right) = \left(p_\infty - p_0\right)\left(\beta + \frac{\nu_0}{p_\infty}\right) + \alpha\frac{x_\infty - x_0}{p_\infty}$$

The long term response is then much different depending if we are below or above the bubble pressure. If we are above, then $p_+ = p_\infty$. If we are under, then we follow Henry's law. The system will then finally fail if there is enough gas to build-up a constant pressure with then will be greater than the confining pressure at some point of the unloading.